TI-85 Graphing Calculator

Most keys on the *TI-85* calculator have more than one function. An orange command/symbol above a key can be activated by pressing (where necessary) the orange `2nd` key first. Similarly, a blue command/symbol above a key can be activated by pressing (where necessary) the blue `ALPHA` key first. Occasionally, a menu will appear on the bottom of the calculator screen. The various menu items are accessed using the `F1` through `F5` keys. Most functions on the *TI-85* can be accessed, if desired, via the `CATALOG` command (`2nd+CUSTOM`).

To adjust the number of decimal places displayed, type `2nd`, then `More`, and use the gray arrow keys to move down to the `Float` line and over to the desired number of decimal places, and then hit `ENTER`. Finally, hit `EXIT` to return to the home screen. In all of Figures D.25 through D.30, we assume that exactly three decimal places are chosen.

There are two types of "−" keys on the calculator: the gray "−" key is used to negate a quantity (a unary operation), while the black "−" key is used for subtracting one quantity from another (a binary operation). To repeat the last input, type `2nd`, then `ENTER`.

In Figures D.25 to D.29, the "Keystrokes" column indicates the exact sequence of keys pressed on the *TI-85* to create each line of input, while the "Input" column illustrates how each line of input actually appears on the calculator screen.

Input of Vectors and Matrices; Fundamental Operations

To enter a vector, hit the `2nd` key and then the "`8`" key. (This opens the `VECTR` menu.) Next, hit `F2` (`EDIT`) and the calculator enters alphanumeric (blue) mode so that you can type in a name using the blue letters above the keys for the vector (names should be no more than 8 characters long), and then hit `ENTER`. Then type in the size of the vector, and a template will appear into which you type the vector entries. (Hit `ENTER` or the "down" arrow key after each entry.) When finished, hit `EXIT` to return to the home screen. A similar process is used to enter a matrix. First, hit the `2nd` key and then the "`7`" key. (This opens the `MATRX` menu.) Then hit `F2` and type a name for the matrix, and then hit `ENTER`. Then type in the dimensions of the matrix (hitting `ENTER` after each dimension), and a template will appear into which you type the matrix entries. (Hit `ENTER` after each entry.) When finished, hit `EXIT`.

Figures D.25 and D.26 illustrate the fundamental operations of dot product, scalar multiplication, matrix multiplication, transpose, and inverse. Assume that the vectors $\mathtt{V} = [5, 7, -4]$ and $\mathtt{W} = [-3, 2, -6]$ have been entered, as well as the matrices

$$\mathtt{M} = \begin{bmatrix} 4 & -1 & 6 & -2 \\ -3 & 2 & -3 & 2 \\ -6 & 8 & 1 & 3 \end{bmatrix} \quad \text{and} \quad \mathtt{N} = \begin{bmatrix} 2 & -3 & 0 \\ 6 & 8 & -1 \\ 3 & 1 & -2 \\ 2 & -4 & -2 \end{bmatrix}.$$

There are various vector and matrix functions/operations listed under the `VECTR` and `MATRX` menus. Figures D.25 and D.26 give the correct sequence of keystrokes for several of these. The `dot` function (in the `MATH` submenu of `VECTR`) calculates the dot product of two vectors. Addition and subtraction are performed using the "+" and (the black) "−" keys, respectively. Scalar multiplication can be implied simply by putting a scalar in front of a vector or matrix. Matrix multiplication is performed using the "×" key, and displayed on the screen as "*". The "^" key is used to find positive integer powers of a square matrix. The orange "$\mathtt{x}^{-1}$" key is used to calculate the inverse of a square matrix. The "`T`" function (in the `MATH` submenu of `MATRX`) calculates the transpose of a matrix.

To assign a name to a vector or matrix result, type `STO→` and the desired name, before hitting the `ENTER` key. (The `STO→` key places the calculator in `ALPHA` (alphanumeric) mode; that is, you do not need to hit the `ALPHA` key before entering a name.) For example, in Figure D.25, the result of the first calculation is stored as the vector `X`, and in Figure D.26, the result of the second calculation is stored as the matrix `P`.

To print out a vector or matrix in fractional form (where possible), use the `►Frac` command (in the `MISC` submenu of the `MATH` menu (above the "×" key)), as shown in Figure D.26 for the matrix `P`.

Keystrokes	**Input**	**Output**
`2,ALPHA,2,+,3,ALPHA, 3,STO→,+,ENTER`	`2V+3W→X`	[1.000 20.000 −26.000]
`ALPHA,+,ENTER`	`X`	[1.000 20.000 −26.000]
`2nd,8,F3,F4,ALPHA,2, comma,ALPHA,3,),ENTER`	`dot(V,W)`	23.000

Figure D.25: *TI-85* session: vectors; fundamental vector operations

Keystrokes	**Input**	**Output**
`ALPHA,8,-,2,ALPHA, 9,2nd,7,F3,F2, ENTER`	`M-2N`$^{\mathrm{T}}$	[[0.000 −13.000 0.000 −6.000] [3.000 −14.000 −5.000 10.000] [−6.000 10.000 5.000 7.000]]
`ALPHA,8,×,ALPHA,9, STO→,comma, ENTER`	`M*N→P`	[[16.000 −6.000 −7.000] [1.000 14.000 0.000] [45.000 71.000 −16.000]]
`ALPHA,comma,2nd, EE,2nd,×, F5, MORE,F1,ENTER`	`P`$^{-1}$ `►Frac`	[[−224/233 − 593/233 98/233] [16/233 59/233 −7/233] [−559/233 −1406/233 230/233]]
`ALPHA,comma,2nd, EE,ENTER`	`P`$^{-1}$	[[−.961 −2.545 .421] [.069 .253 −.030] [−2.399 −6.034 .987]]

Figure D.26: *TI-85* session: matrices; fundamental matrix operations

To delete a vector or matrix from the calculator memory, type `2nd,+(MEM),F2(DELETE)`. Then type `F5` to delete a vector or type `MORE,F1` to delete a matrix. Finally, use the "down" arrow key to move the cursor to the desired vector or matrix to be deleted, and then hit `ENTER` followed by `EXIT`.

Solving a Linear System; Gauss-Jordan Row Reduction Method

The `rref` function (in the `OPS` submenu of the `MATRX` menu) calculates the reduced row echelon form of a (possibly augmented) matrix. Assume for Figure D.27 that the matrix

$$\texttt{R} = \begin{bmatrix} 3 & 1 & 7 & 2 & 13 \\ 2 & -4 & 14 & -1 & -10 \\ 5 & 11 & -7 & 8 & 59 \\ 2 & 5 & -4 & -3 & 39 \end{bmatrix}$$

has been entered into the calculator. `R` is the augmented matrix for a linear system with an infinite solution set. From the result in Figure D.27, you can see that the general solution set of this system is $\{(-3c+4,\, 2c+5,\, c,\, -2)\}$.

Keystrokes	Input	Output
`2nd,7,F4,F5,` `ALPHA,5,` `ENTER`	`rref R`	[[1.000 0.000 3.000 0.000 4.000] [0.000 1.000 −2.000 0.000 5.000] [0.000 0.000 0.000 1.000 −2.000] [0.000 0.000 0.000 0.000 0.000]]

Figure D.27: *TI-85* session: solution of a linear system; row reduction

If a linear system has a nonsingular coefficient matrix, and hence has a unique solution, you can solve the system using the orange `SIMULT` function. `SIMULT` works for any linear system having no more than 30 equations or 30 unknowns. This function asks you to first type in the number of linear equations in the system, and a template appears in which you can enter the coefficients of each row in turn.

Determinants, Eigenvalues/Eigenvectors

The `det` function (under the `MATH` submenu of the `MATRX` menu) calculates the determinant of a square matrix. Eigenvalues for a given square matrix can be found using the `eigVl` function. On the *TI-85*, the `eigVc` function returns a matrix whose columns are eigenvectors for a given square matrix. These functions are illustrated in Figure D.28, for the matrix

$$\texttt{T} = \begin{bmatrix} 5 & 2 & 0 & 1 \\ -2 & 1 & 0 & -1 \\ 4 & 4 & 3 & 2 \\ 16 & 0 & -8 & -5 \end{bmatrix},$$

which has two eigenvalues, $\lambda_1 = -5$, having algebraic and geometric multiplicity 1, and $\lambda_2 = 3$, having algebraic multiplicity 3 and geometric multiplicity 2. Assume that matrix `T` has already been entered into the calculator.

Keystrokes	**Input**	**Output**
`2nd,7,F3,F1,` `ALPHA,-,ENTER`	`det T`	-135.000
`2nd,7,F3,F4,` `ALPHA,-,ENTER`	`eigVl T`	{(3.000, 1.836E−6), (3.000, −1.836E−6), (−5.000, 0.000), (3.000, 0.000)}
`2nd,7,F3,F5,` `ALPHA,-,ENTER`	`eigVc T`	[[(7.124E−4, −25.812) (7.124E−4, 25.812) [(−7.006E−4, 25.812) (−7.124E−4, −25.812) [.001, −51.623) (.001, 51.623) [(2.369E−5, 6.522E−10) (2.369E−5, −6.522E−10) (−.320, 0.000) (.681, 0.000)] (.320, 0.000) (−.181, 0.000)] (−.640, 0.000) (2.361, 0.000)] (2.562, 0.000) (−1.000, 0.000)]]

Figure D.28: *TI-85* session: eigenvalues and eigenvectors

The eigenvalues and the eigenvector entries are expressed as ordered pairs because they are written as complex numbers, where the first entry of each ordered pair is the real part, and the second entry is the imaginary part (see Appendix C of the textbook). That is, an ordered pair (a, b) in the output represents the complex number $a + bi$. The notation "E$-k$" indicates that the immediately preceding number should be multiplied by 10^{-k}. If k is large, these numbers are extremely small. Therefore, all of the entries in Figure D.28 containing "E" are zero, for all practical purposes (as is the entry .001).

Thus, we can use the results of the `eigVc` function on the *TI-85* to get a set of eigenvectors for `T`: $\{[-25.812i,\ 25.812i,\ -51.623i,\ 0],\ [-.320,\ .320,\ -.640,\ 2.562],\ [.681,\ -.181,\ 2.361,\ -1]\}$. The first eigenvector corresponds to the eigenvalue $\lambda_2 = 3$ and is derived from the first two columns of the *TI-85* output in Figure D.28. (The second column is the negative of the first column.) The second eigenvector corresponds to $\lambda_1 = -5$ and is derived from the third column. The third eigenvector corresponds to $\lambda_2 = 3$ and comes from the fourth column of this output. You should verify that these are indeed eigenvectors for `T`. Note that while the eigenvector obtained from the first two columns contains complex entries, it can be multiplied by i to yield the real eigenvector $[25.812,\ -25.812,\ 51.623,\ 0]$ for $\lambda_2 = 3$.

Of course, a basis of eigenvectors for each eigenvalue λ of a (square) matrix `A` can also be calculated by row reducing the matrix $\lambda\mathbf{I}_n -$ `A`, setting each independent variable in turn equal to 1 with all others equal to 0, and then solving for the dependent variables. The function `ident` k (`ident` is in the `OPS` submenu of the `MATRX` menu) creates a $k \times k$ identity matrix.

These operations are illustrated in Figure D.29. Linearly independent eigenvectors for the earlier matrix T, for eigenvalue $\lambda_2 = 3$, are found in Figure D.29 from the reduced row echelon form matrix R for $\mathtt{S} = 3\mathbf{I}_4 - \mathtt{T}$. First, by letting the third column variable of R equal 1 and its fourth column variable equal 0, we obtain $[\frac{1}{2}, -\frac{1}{2}, 1, 0]$, and then by letting its third column variable equal 0 and fourth column variable equal 1, we obtain $[\frac{1}{2}, -1, 0, 1]$. Although it is not readily apparent, it can be shown that the set $\{[\frac{1}{2}, -\frac{1}{2}, 1, 0], [\frac{1}{2}, -1, 0, 1]\}$ spans the same two-dimensional subspace of $\mathbb{R}^4$ as the set $\{[25.812, \ -25.812, \ 51.623, \ 0], [.681, \ -.181, \ 2.361, \ -1]\}$ of eigenvectors for $\lambda_2 = 3$ obtained earlier from eigVc (ignoring error due to roundoff). For example, $[\frac{1}{2}, -\frac{1}{2}, 1, 0]$ by 25.812 and rounding to three significant digits produces $[25.812, \ -25.812, \ 51.623, \ 0]$.

Keystrokes	Input	Output
2nd,7,F4,F3, 4,STO→,), ENTER	ident 4 → I	$\begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 \end{bmatrix}$
3,×,ALPHA,),-, ALPHA,-,STO→, 6,ENTER	3*I-T→S	$\begin{bmatrix} -2.000 & -2.000 & 0.000 & -1.000 \\ 2.000 & 2.000 & 0.000 & 1.000 \\ -4.000 & -4.000 & 0.000 & -2.000 \\ -16.000 & 0.000 & 8.000 & 8.000 \end{bmatrix}$
2nd,7,F4,F5, ALPHA,6,STO→, 5,ENTER	rref S → R	$\begin{bmatrix} 1.000 & 0.000 & -.500 & -.500 \\ 0.000 & 1.000 & .500 & 1.000 \\ 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 \end{bmatrix}$

Figure D.29: *TI-85* session: direct calculation of eigenspace

Gram-Schmidt Process

Assume the vectors $\mathtt{C} = [2, 1, 0, -1]$, $\mathtt{D} = [1, 0, 2, -1]$, and $\mathtt{E} = [0, -2, 1, 0]$ have already been stored in the calculator. In Figure D.30, we perform a Gram-Schmidt Process to create an orthogonal basis for the span of these vectors in $\mathbb{R}^4$. An orthonormal basis for the span can easily be produced by dividing each orthogonal basis vector by its length. The norm function (under the MATH submenu of the VECTR menu) calculates the length of a given vector.

Input	Output
`D - (dot(D,C) / dot(C,C))*C` $\rightarrow$ `F`	$[0.000 \quad -.500 \quad 2.000 \quad -.500]$
`E - (dot(E,C) / dot(C,C))*C - (dot(E,F) / dot(F,F))*F` $\rightarrow$ `G`	$[.667 \quad -1.333 \quad -.333 \quad 1.000\mathrm{E}-14]$
`C / norm(C)` $\rightarrow$ `J`	$[.816 \quad .408 \quad 0.000 \quad -.408]$
`F / norm(F)` $\rightarrow$ `K`	$[0.000 \quad -.236 \quad .943 \quad -.236]$
`G / norm(G)` $\rightarrow$ `L`	$[.436 \quad -.873 \quad -.218 \quad 6.547\mathrm{E}-15]$

Figure D.30: *TI-85* session: Gram-Schmidt Process; orthogonal and orthonormal bases

You can easily verify that $\{[2,1,0,-1],\ [0,-\frac{1}{2},2,-\frac{1}{2}],\ [\frac{2}{3},-\frac{4}{3},-\frac{1}{3},0]\}$ (vectors `C`, `F`, `G`) is an orthogonal set of vectors spanning the same subspace of $\mathbb{R}^4$ as $\{[2,1,0,-1],\ [1,0,2,-1],\ [0,-2,1,0]\}$. An orthonormal basis for the same subspace is given by the set of vectors $\{$`J`, `K`, `L`$\}$.