Derive for Windows, Version 4.10

Algebraic expressions in *Derive 4 for Windows* must be "authored" before operators can be applied to them. This is easy to do: simply click on `Author` on the menu bar, then click on `Expression` and type in the desired expression. Alternatively, you can use the icon on the toolbar corresponding to the `Author, Expression` commands. In fact, the toolbar contains shortcuts for many of the most frequently used commands.

Derive 4 for Windows numbers each line of input and output as it is created on the screen. The first line is referred to as "#1," the second as "#2," and so on. The line numbering system in *Derive 4 for Windows* makes it easy to refer to any previously created expression, as you will observe in the accompanying figures.

At any given time in *Derive 4 for Windows*, at least one line or part of a line is highlighted. You can change the highlighted area by using the arrow keys to scroll up or down. Some commands, such as `Simplify`, are designed to apply to the currently highlighted expression. In *Derive 4 for Windows* the most recent output line is highlighted by default, which makes it convenient to simplify the most recently computed expression. For example, if line #6 contains a dot product such as $[2, -3, 4] \cdot [-1, 0, -2]$, and if this dot product is highlighted on the screen, then clicking on `Simplify` and then on `Basic` produces a new line (#7) containing the dot product -10.

Input of Vectors and Matrices; Fundamental Operations

To enter a vector or matrix, simply click on `Author`, and then on `Vector` or `Matrix`. After indicating the dimension(s) of the vector or matrix, a template of the correct size will form, which you fill in with the desired vector or matrix entries. Use the `Tab` key to move from entry to entry.

Use the "+" and "−" keys between vectors or matrices for addition and subtraction, respectively. Place a scalar before a vector or matrix to perform scalar multiplication. Use a "`.`" (period) between vectors to perform dot product, and between matrices to perform matrix multiplication. Use the " ‘ " (left apostrophe) symbol after a matrix to calculate its transpose. The "`^`" symbol is used to find powers of a (square) matrix. In particular, use "`^`(−1)" to find the inverse of a (square) matrix. To convert fractions to decimals, use the `Approximate` command, and specify the desired number of significant digits displayed. These operations are illustrated in Figures D.7 and D.8.

Input	Output
`Author,Vector,3,OK,5,Tab,7,Tab,-4,OK`	#1: $[5, 7, -4]$
`Author,Vector,3,OK,x,Tab,y,Tab,z,OK`	#2: $[x, y, z]$
`Author,Expression,2#1+3#2,Simplify`	#3: $[3x + 10,\ 3y + 14,\ 3z - 8]$
`Author,Expression,#1.#2,Simplify`	#4: $5x + 7y - 4z$

Figure D.7: *Derive 4 for Windows* session: vectors; fundamental vector operations

Input	Output
`Author,Matrix,3,4,OK,` `4,Tab,-1,Tab,6,Tab,-2,Tab,` `-3,Tab,2,Tab,-3,Tab,2,Tab,` `-6,Tab,8,Tab,1,Tab,3,OK`	#5: $\begin{bmatrix} 4 & -1 & 6 & -2 \\ -3 & 2 & -3 & 2 \\ -6 & 8 & 1 & 3 \end{bmatrix}$
`Author,Matrix,4,3,OK,2,Tab,-3,`(etc.)	#6: $\begin{bmatrix} 2 & -3 & 0 \\ 6 & 8 & -1 \\ 3 & 1 & -2 \\ 2 & -4 & -2 \end{bmatrix}$
`Author,Expression,#5-2(#6‘),Simplify`	#7: $\begin{bmatrix} 0 & -13 & 0 & -6 \\ 3 & -14 & -5 & 10 \\ -6 & 10 & 5 & 7 \end{bmatrix}$
`Author,Expression,#5.#6,Simplify`	#8: $\begin{bmatrix} 16 & -6 & -7 \\ 1 & 14 & 0 \\ 45 & 71 & -16 \end{bmatrix}$
`Author,Expression,#8^(-1),Simplify`	#9: $\begin{bmatrix} -\frac{224}{233} & -\frac{593}{233} & \frac{98}{233} \\ \frac{16}{233} & \frac{59}{233} & -\frac{7}{233} \\ -\frac{559}{233} & -\frac{1406}{233} & \frac{230}{233} \end{bmatrix}$
`Simplify,Approximate,4,Approximate`	#10: $\begin{bmatrix} -0.9613 & -2.545 & 0.4206 \\ 0.06866 & 0.2532 & -0.03004 \\ -2.399 & -6.034 & -0.9871 \end{bmatrix}$

Figure D.8: *Derive 4 for Windows* session: matrices; fundamental matrix operations

Solving a Linear System; Gauss-Jordan Row Reduction Method

You can solve a linear system directly using the `Solve` command. Click on `Solve` and then `System`, and then indicate the number of equations to be solved.

A template will appear, into which you type each equation in turn. Under the template, indicate the variables to be solved for. If the system has no solution, the result "`[ ]`" will appear. If the system has more than one solution, each independent variable will be represented with a differently numbered "@" symbol, and the dependent variables will be expressed in terms of the independent ones.

You can also solve a system using the `row_reduce` function. This function computes the reduced row echelon form of a (possibly augmented) matrix. The matrix on output line #12 below is the augmented matrix for a linear system with an infinite solution set.

In Figure D.9, the same linear system is solved using both `Solve` and `row_reduce`. You should verify that the general solution set obtained from either method is equivalent to $\{(-3c+4,\ 2c+5,\ c,\ -2)\}$.

Input	Output
`Solve,System,4,OK,3x+y+7z+2w =13,Tab,2x-4y+14z-w=-10, Tab,5x+11y-7z+8w=59,Tab, 2x+5y-4z-3w=39,Tab` (`x, y, z, w` are highlighted),`Simplify`	#11: $x = @1,\ y = \frac{23-2\cdot @1}{3},\ z = \frac{4-@1}{3},\ w = -2$
`Author,Matrix,4,5,OK,3,Tab,`(etc.)	#12: $\begin{bmatrix} 3 & 1 & 7 & 2 & 13 \\ 2 & -4 & 14 & -1 & -10 \\ 5 & 11 & -7 & 8 & 59 \\ 2 & 5 & -4 & -3 & 39 \end{bmatrix}$
`Author`, `Expression`, `row_reduce`(#12), `Simplify`	#13: $\begin{bmatrix} 1 & 0 & 3 & 0 & 4 \\ 0 & 1 & -2 & 0 & 5 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Figure D.9: *Derive 4 for Windows* session: solution of a linear system; row reduction

Determinants; Eigenvalues/Eigenvectors; Characteristic Polynomial

The `det` function calculates the determinant of a (square) matrix. If `M` is an $n \times n$ matrix, where n is even, the `charpoly(M,x)` function computes the characteristic polynomial for `M`, using `x` as the variable. In *Derive 4 for Windows*, the `charpoly(M,x)` function actually returns the determinant $|\mathbf{M} - x\mathbf{I}_n|$, which is $(-1)^n|x\mathbf{I}_n - \mathbf{M}|$. Thus, for n odd, *Derive 4 for Windows* computes the negative of the characteristic polynomial as defined in the textbook.[1] The function `eigenvalues(M,x)` calculates the eigenvalues for `M`. Finally, the functions `exact_eigenvector(M,`λ`)` and `approx_eigenvector(M,`λ`)` give a general

[1]Some linear algebra texts use $|\mathbf{M} - x\mathbf{I}_n|$ to define the characteristic polynomial of $\mathbf{M}$ rather than $|x\mathbf{I}_n - \mathbf{M}|$.

eigenvector of `M` for eigenvalue λ. Use the former function when the exact value of λ is known, but the latter when only a close approximation of λ is known. (Note: The eigenvector functions are in a utility file known as "vector.mth". This file can be loaded by clicking on `File`, then `Load`, then `Utility`.)

Most of these functions are illustrated in Figure D.10, where the matrix in output line #14 has two eigenvalues, $\lambda_1 = -5$, having algebraic and geometric multiplicity 1, and $\lambda_2 = 3$, having algebraic multiplicity 3 and geometric multiplicity 2. Output line #18 displays a general eigenvector for the (exact) eigenvalue -3 of the matrix in line #14. Notice that the independent variables in line #18 are denoted "@2" and "@3" since an independent variable "@1" was already labeled in line #13. Letting "@2" equal 1, and "@3" equal 0, we obtain the eigenvector $[1, 0, 4, -2]$, while letting "@2" equal 0, and "@3" equal 1, we obtain the eigenvector $[0, 1, 2, -2]$. These two eigenvectors form a basis in $\mathbb{R}^4$ for the eigenspace E_3.

Input	**Output**
`Author,Matrix,4,4,OK,5,Tab,2,`(etc.)	#14: $\begin{bmatrix} 5 & 2 & 0 & 1 \\ -2 & 1 & 0 & -1 \\ 4 & 4 & 3 & 2 \\ 16 & 0 & -8 & -5 \end{bmatrix}$
`Author,Expression,charpoly(#14,x),Simplify`	#15: $x^4 - 4x^3 - 18x^2$ $+108x - 135$
`Simplify,Factor` (with default `Rational`)`,Factor`	#16: $(x+5)(x-3)^3$
`Author,Expression,eigenvalues(#14,x),Simplify`	#17: $[x = 3,\ x = -5]$
`File,Load,Utility` (select: `vector.mth`)`,Open`	("vector.mth loaded")
`Author,Expression,exact_eigenvector(#14,3)`	#18 : $x1 = @2, \quad x2 = @3,$ $x3 = 2 \cdot (2@2 + @3),$ $x4 = -2 \cdot (@2 + @3)$

Figure D.10: *Derive 4 for Windows* session: characteristic polynomial; eigenvalues; eigenvectors

The characteristic polynomial can also be computed directly using determinants. The $n \times n$ identity matrix is created using the function `identity_matrix(`n`)`.

If output line #k consists of a square matrix, a basis of eigenvectors for any eigenvalue λ of the matrix can also be calculated by row reducing the matrix "$\lambda \mathbf{I}_n - \#k$", setting each independent variable in turn equal to 1 with all others equal to 0, and then solving for the dependent variables.

These operations are illustrated in Figure D.11 (see next page) for the matrix in output line #14. Linearly independent eigenvectors for the eigenvector $\lambda_2 =$

3 are found from the reduced row echelon form matrix for $3\mathbf{I}_4 - \#14$, given in output line #21. Letting the third column variable equal 1 for the matrix in output line #21, and its fourth column variable equal 0, we obtain $[\frac{1}{2}, -\frac{1}{2}, 1, 0]$, and letting its third column variable equal 0 and fourth column variable equal 1, we obtain $[\frac{1}{2}, -1, 0, 1]$. (You can easily verify that $\{[\frac{1}{2}, -\frac{1}{2}, 1, 0], [\frac{1}{2}, -1, 0, 1]\}$ spans the same two-dimensional subspace of $\mathbb{R}^4$ as the set $\{[1, 0, 4, -2], [0, 1, 2, -2]\}$ of eigenvectors obtained earlier from the `exact_eigenvector` function.)

Gram-Schmidt Process

The necessary calculations for the Gram-Schmidt Process can be performed in *Derive 4 for Windows* in the manner illustrated in Figure D.12. We begin with a given set of three linearly independent vectors $\{[2, 1, 0, -1], [1, 0, 2, -1], [0, -2, 1, 0]\}$ in $\mathbb{R}^4$ (output lines #22 through #24), and construct an orthogonal basis for the span of those vectors. We then produce an orthonormal basis for the span by dividing each vector in the orthogonal basis by its length. The `abs` function calculates the length of a given vector.

Input	Output
`Author,Expression,` `det(x*identity_matrix(4)-#14),Simplify`	#19: $x^4 - 4x^3 - 18x^2 + 108x - 135$
`Author,Expression,` `3*identity_matrix(4)-#14,Simplify`	#20: $\begin{bmatrix} -2 & -2 & 0 & -1 \\ 2 & 2 & 0 & 1 \\ -4 & -4 & 0 & -2 \\ -16 & 0 & 8 & 8 \end{bmatrix}$
`Author,Expression,` `row_reduce(#20),Simplify`	#21: $\begin{bmatrix} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Figure D.11: *Derive 4 for Windows* session: characteristic polynomial via determinant; direct calculation of eigenspace

You can easily verify that $\{[2, 1, 0, -1], [0, -\frac{1}{2}, 2, -\frac{1}{2}], [\frac{2}{3}, -\frac{4}{3}, -\frac{1}{3}, 0]\}$ (output lines #22, #25, and #26) is an orthogonal set of vectors spanning the same subspace of $\mathbb{R}^4$ as $\{[2, 1, 0, -1], [1, 0, 2, -1], [0, -2, 1, 0]\}$. An orthonormal basis for the same subspace is given by the vectors in output lines #27, #28, and #29.

Input	Output
`Author,Vector,4,OK,2,Tab,1,` `Tab,0,Tab,-1,OK`	#22: $[2, 1, 0, -1]$
`Author,Vector,4,OK,1,Tab,0,` `Tab,2,Tab,-1,OK`	#23: $[1, 0, 2, -1]$
`Author,Vector,4,OK,0,Tab,-2,` `Tab,1,Tab,0,OK`	#24: $[0, -2, 1, 0]$
`Author,Expression,` `#23-(#23.#22)/(#22.#22)#22,Simplify`	#25: $[0, -\frac{1}{2}, 2, -\frac{1}{2}]$

`Author,Expression,`
`#24-(#24.#22)/(#22.#22)#22`
`-(#24.#25)/(#25.#25)#25,Simplify`
	#26: $[\frac{2}{3}, -\frac{4}{3}, -\frac{1}{3}, 0]$

`Author,Expression,#22/abs(#22),Simplify`	#27: $\left[\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}, 0, -\frac{\sqrt{6}}{6}\right]$
`Author,Expression,#25/abs(#25),Simplify`	#28: $\left[0, -\frac{\sqrt{2}}{6}, \frac{2\sqrt{2}}{3}, -\frac{\sqrt{2}}{6}\right]$
`Author,Expression,#26/abs(#26),Simplify`	#29: $\left[\frac{2\sqrt{21}}{21}, -\frac{4\sqrt{21}}{21}, -\frac{\sqrt{21}}{21}, 0\right]$

Figure D.12: *Derive 4 for Windows* session: Gram-Schmidt Process; orthogonal and orthonormal bases