MAT 2361 Logic and Foundations
Record of Assignments
Spring 2009

Assignment 1.
Handout, numbers 1 through 4.

Due 1/26.

Assignment 2.
Handout, numbers 5 through 24.

Due 2/2.

Assignment 3.

Due 2/9.

Hamilton p10: 3bc, 5ab, 6ab
Hamilton p15: 9a

Extra problem#1 ("Disjunctive Normal Form").

Let $\{p_1, p_2, \ldots p_n\}$ be proposition letters, and consider a truth table with n columns (one for each p) and 2^n rows, one for each possible sequence of n Ts and Fs. If you make another column and randomly fill in Ts and Fs, you are making a truth function like the ones we made in class. This exercise will show that regardless what function you make, there is a proposition in the letters Let $\{p_1, p_2, \ldots p_n\}$ that has that truth function.

[**a**]. Show that if every letter you filled in was an F, then $p_1 \wedge (\sim p_1)$ is a proposition with this truth function.

[**b**]. Show that if the function you filled in has one T and the rest Fs, it is the truth table of a proposition of the form

$$C := q_1 \wedge q_2 \wedge \cdots \wedge q_n, \tag{1}$$

where each q_i is either p_i or $(\sim p_i)$.

[**c**]. Show that if the function you filled in has k Ts and the rest Fs (where $2 \leq k \leq 2^n$), then it is the truth table of a proposition of the form

$$D := C_1 \vee C_2 \vee \cdots \vee C_j \vee \cdots \vee C_k,$$

where each C_j is of the form you got in part [**b**].

Assignment 4.

Due 2/16.

Hamilton P36: 2ab, 3ab, 5.
Hint for 2b: I found easier to establish directly that $\vdash_L (\sim\sim \mathcal{A} \longrightarrow \mathcal{A})$ and then to use the converse of the Deduction Theorem.
Hint for 3b: First make subsidiary deduction $(\mathcal{B} \longrightarrow \mathcal{A}) \vdash_L (\sim \mathcal{A} \longrightarrow \sim \mathcal{B})$. My solution uses result in problem 2b twice and uses HS twice.

(assgt. 4 continues⟶)

Extra problem#2.

Suppose that there is a contradiction in L; that is, there is a formula $\mathcal{A}$ for which

$$\begin{cases} \vdash_L \mathcal{A} \\ \text{and} \\ \vdash_L \sim\mathcal{A}. \end{cases}$$

Show that $\vdash_L \mathcal{B}$ for *every* formula $\mathcal{B}$.

Assignment 5.

Due 2/23.

Exercises 1 and 2 on the handout titled *The Completeness of the Propositional Calculus*.

Extra Problem#3: A deduction rule for the Propositional Calculus.

Using the now-established fact that every tautology is a theorem of L, establish the following deduction rule in L.

Proposition. *Let Γ be a sequence of wfs and let $\mathcal{A}$ and $\mathcal{B}$ be wfs. If*

$$\begin{cases} \Gamma \oplus \{\mathcal{A}\} \vdash_L \mathcal{B} \\ \text{and} \\ \Gamma \oplus \{\mathcal{A}\} \vdash_L \sim\mathcal{B}, \end{cases}$$

then

$$\Gamma \vdash_L \sim\mathcal{A}.$$

Hamilton P44: 7,8

Extra Problem#4. Closure properties of "Thm(·)".

As in class: Let $\mathcal{W}$ denote the set of all *wf*s of L, and for any subset $S \subseteq \mathcal{W}$, let $\text{Thm}(S)$ be the set of all *wf*s in $\mathcal{W}$ which are deducible from S. (Equivalently: $\text{Thm}(S)$ is the intersection of all subsets T of $\mathcal{W}$ such that $S \subseteq T$ and T is closed under the operation *modus ponens*.)

Show, for any subsets S_1 and S_2 of $\mathcal{W}$:

- **[a].** $S_1 \subseteq \text{Thm}(S_1)$.
- **[b].** $S_1 \subseteq S_2 \implies \text{Thm}(S_1) \subseteq \text{Thm}(S_2)$.
- **[c].** $\text{Thm}(\text{Thm}(S_1)) = \text{Thm}(S_1)$.

Assignment 6.

Due 3/3.

Handout on Agreement/Substitution theorems: exercises 1–6.
Hamilton P56: 4,5.

Try **without writing up or submitting** all parts of Hamilton P56 numbers 6–10 for which the author provides a solution (P 206).

Assignment 7.

Due 3/24

Hamilton: P59, 11; P69, 17bd, 19b.
Hamilton: P80, 1, 2, 3a.
Note: In 2(b), the text assumes that $\mathcal{B}$ does not contain x_i free; in fact, you must assume this for 2(a) as well.
$\mathcal{T}$ handout, exercises 1–3.

Assignment 8.

Due 4/7

Adequacy Theorem Handout: Exercises 2–8.
Hamilton: P85, 5.
Hamilton: P100, 13–15.
Hamilton: P103, 17, 20.

Assignment 9.

Due 4/21

Numeralwise Expressibility Handout, Exercises 1–7.