An Example of the Deduction Theorem at Work

The example below illustrates the constructive nature of the proof of the Deduction Theorem. From the subsidiary deduction at the left, the steps of the proof construct the resulting deduction at the right. Each line of the subsidiary deduction corresponds to the line immediately to its right: when formula $\mathcal{X}$ appears on the left, then the formula $(\sim\mathcal{B} \to \mathcal{X})$ appears at the right.

$\sim\mathcal{B} \underset{L}{\vdash} (\mathcal{B} \to \mathcal{A})$

1.	$\sim\mathcal{B} \to (\sim\mathcal{A} \to\sim\mathcal{B})$	$L1$
2.	$\sim\mathcal{B}$	As.
3.	$\sim\mathcal{A} \to\sim\mathcal{B}$	MP(2,1)
4.	$(\sim\mathcal{A} \to\sim\mathcal{B}) \to (\mathcal{B} \to \mathcal{A})$	$L3$
5.	$\mathcal{B} \to \mathcal{A}$	MP(3,4)

$\underset{L}{\vdash} \sim\mathcal{B} \to (\mathcal{B} \to \mathcal{A})$

1.	$\sim\mathcal{B} \to (\sim\mathcal{A} \to\sim\mathcal{B})$	$L1$
2.	$(\sim\mathcal{B} \to (\sim\mathcal{A} \to\sim\mathcal{B})) \to \big(\sim\mathcal{B} \to (\sim\mathcal{B} \to (\sim\mathcal{A} \to\sim\mathcal{B}))\big)$	$L1$
3.	$\sim\mathcal{B} \to (\sim\mathcal{B} \to (\sim\mathcal{A} \to\sim\mathcal{B}))$	MP(1,2)
4.	$\sim\mathcal{B} \to\sim\mathcal{B}$	Theorem of L
5.	$\big(\sim\mathcal{B} \to (\sim\mathcal{B} \to (\sim\mathcal{A} \to\sim\mathcal{B}))\big) \to$ $(\sim\mathcal{B} \to\sim\mathcal{B}) \to (\sim\mathcal{B} \to (\sim A \to\sim\mathcal{B}))$	$L2$
6.	$(\sim\mathcal{B} \to\sim\mathcal{B}) \to (\sim\mathcal{B} \to (\sim A \to\sim\mathcal{B}))$	MP(3,5)
7.	$\sim\mathcal{B} \to (\sim A \to\sim\mathcal{B})$	MP(4,6)
8.	$\big((\sim\mathcal{A} \to\sim\mathcal{B}) \to (\mathcal{B} \to \mathcal{A})\big) \to$ $\big(\sim\mathcal{B} \to \big((\sim\mathcal{A} \to\sim\mathcal{B}) \to (\mathcal{B} \to \mathcal{A})\big)\big)$	$L1$
9.	$(\sim\mathcal{A} \to\sim\mathcal{B}) \to (\mathcal{B} \to \mathcal{A})$	$L3$
10.	$\sim\mathcal{B} \to ((\sim\mathcal{A} \to\sim\mathcal{B}) \to (\mathcal{B} \to \mathcal{A}))$	MP(9,8)
11.	$\big(\sim\mathcal{B} \to ((\sim\mathcal{A} \to\sim\mathcal{B}) \to (\mathcal{B} \to \mathcal{A}))\big) \to$ $\big((\sim\mathcal{B} \to (\sim\mathcal{A} \to\sim\mathcal{B})) \to (\sim\mathcal{B} \to (\mathcal{B} \to \mathcal{A}))\big)$	$L2$
12.	$(\sim\mathcal{B} \to (\sim\mathcal{A} \to\sim\mathcal{B})) \to (\sim\mathcal{B} \to (\mathcal{B} \to \mathcal{A}))$	MP(10,11)
13.	$\sim\mathcal{B} \to (\mathcal{B} \to \mathcal{A})$	MP(7,12)