Complex Treasure Hunt

Rachel Hall • Complex Analysis • April 30th, 2004

Directions Ideally, I'd like to get only one paper from each team, but if you can't get together, give me what you have separately. The team with the highest score gets two points each; the second place team gets one point each. The scoring is

- +1 for each correct answer—that is, at least one person turns in the correct answer and no one turns in an incorrect answer.
- −1/2 for each incorrect answer. If I receive multiple solutions from the same team and one of them is incorrect, that problem counts as incorrect.

For each question, find a function that matches that description.

1. My derivative is $e^{-2\pi iz}$.
2. My antiderivative is $e^{-2\pi iz}$.
3. I am entire.
4. I am not analytic anywhere on the complex plane.
5. I am multiple-valued.
6. The point $z = 2i$ is my only singular point.
7. My only singular points are the sixth roots of unity.
8. I am continuous as a map from the Riemann sphere to the Riemann sphere, but I am not entire as a function on the complex plane.
9. I am discontinuous on the negative real axis and at 0, but I am analytic everywhere else.
10. I map the imaginary axis onto the unit circle (make sure you understand the technical definition of "onto").
11. I am equal to my own modulus (i.e. $f(z) = |f(z)|$ for all z).
12. I map the whole complex plane onto the positive real axis, plus 0 (i.e. onto $\{x + iy \,|\, y = 0 \text{ and } x \geq 0\}$.
13. I have branches and you use me to find roots and do "twig."
14. I map the complex plane minus the nonpositive reals onto the right half-plane $\{z \,|\, \mathrm{Re}\, z > 0\}$.
15. I sound like I should be really easy.
16. I also sound like I should be really easy but I'm not your previous answer.

17. I map the complex plane onto the complex plane, but I am not analytic anywhere.

18. I map the punctured plane $C - \{0\}$ onto the unit circle, and Bill likes me.

19. I'm what you do if you find a burglar in your house and are carrying a large stick.

20. My integral around every closed curve in the complex plane is 0.

21. My integral around the contour $2 + e^{2\pi i t}$ where $0 \leq t \leq 1$ is $2\pi i$.

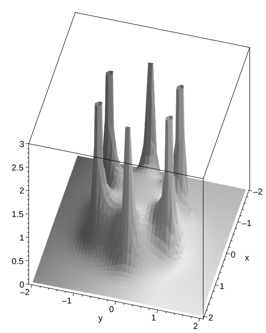
22. My integral around any simple closed curve enclosing the origin equals 1.

23. My integral along the contour $z_1(t) = t$ where $-3 \leq t \leq 3$ equals my integral along the contour $z_2(t) = 3e^{-it}$ where $-\pi \leq t \leq 0$.

24. My Maclaurin series converges everywhere on the complex plane.

25. My Maclaurin series converges when $|z| < 7$ and diverges when $|z| > 7$.

26. My modular surface looks like this:



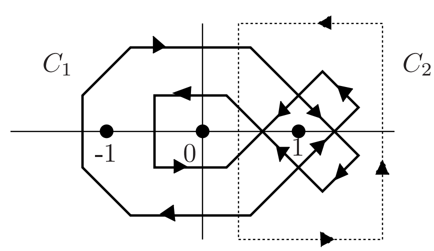
27. I am entire and my maximum modulus is 5 (i.e. $\max(|f(z)|) = 5$).

28. My only singular points are 0, -1, and 1.

29. My only singular points are 0, -1, and 1. My integral around C_2 (see diagram below) is nonzero.

30. My only singular points are 0, -1, and 1. My integral around the curve C_1 is 0.

31. My only singular points are 0, -1, and 1. My integral around the curve C_1 is 0 and my integral around the rectangle C_2 is nonzero.



32. I am your favorite function (awww...).