Let $M = n\ell$, and let D_M act on $S_\ell^n(r)$. The number of orbits is

$$
\begin{cases}
\dfrac{1}{2M}\left[\displaystyle\sum_{\substack{d \mid \gcd(n,r)\\ \gcd(d,\ell)=1}} \phi(d)\binom{n/d}{r/d}\ell^{r/d} + M\binom{\frac{n-1}{2}}{\frac{r}{2}}\ell^{\frac{r}{2}}\right], & \text{if } n \text{ is odd and } r \text{ is even;}\\[3ex]
\dfrac{1}{2M}\left[\displaystyle\sum_{\substack{d \mid \gcd(n,r)\\ \gcd(d,\ell)=1}} \phi(d)\binom{n/d}{r/d}\ell^{r/d} + M\binom{\frac{n-1}{2}}{\frac{r-1}{2}}\ell^{\frac{r-1}{2}}\right], & \text{if } n \text{ is odd and } r \text{ is odd;}\\[3ex]
\dfrac{1}{2M}\left[\displaystyle\sum_{\substack{d \mid \gcd(n,r)\\ \gcd(d,\ell)=1}} \phi(d)\binom{n/d}{r/d}\ell^{r/d} + \frac{M}{2}\left[2\ell\binom{\frac{n-2}{2}}{\frac{r}{2}} + \binom{\frac{n-2}{2}}{\frac{r-2}{2}}\right]\ell^{\frac{r-2}{2}}\right], & \text{if } n \text{ is even, } \ell \text{ is odd, and } r \text{ is even;}\\[3ex]
\dfrac{1}{2M}\left[\displaystyle\sum_{\substack{d \mid \gcd(n,r)\\ \gcd(d,\ell)=1}} \phi(d)\binom{n/d}{r/d}\ell^{r/d} + M\binom{\frac{n-2}{2}}{\frac{r-1}{2}}\ell^{\frac{r-1}{2}}\right], & \text{if } n \text{ is even, } \ell \text{ is odd, and } r \text{ is odd;}\\[3ex]
\dfrac{1}{2M}\left[\displaystyle\sum_{\substack{d \mid \gcd(n,r)\\ \gcd(d,\ell)=1}} \phi(d)\binom{n/d}{r/d}\ell^{r/d} + \frac{M}{2}\left[\binom{\frac{n-2}{2}}{\frac{r}{2}} + \binom{\frac{n}{2}}{\frac{r}{2}}\right]\ell^{\frac{r}{2}}\right], & \text{if } n \text{ is even, } \ell \text{ is even, and } r \text{ is even;}\\[3ex]
\dfrac{1}{2M}\left[\displaystyle\sum_{\substack{d \mid \gcd(n,r)\\ \gcd(d,\ell)=1}} \phi(d)\binom{n/d}{r/d}\ell^{r/d} + M\binom{\frac{n-2}{2}}{\frac{r-1}{2}}\ell^{\frac{r-1}{2}}\right], & \text{if } n \text{ is even, } \ell \text{ is even, and } r \text{ is odd.}
\end{cases}
$$

The new ingredients in the formulas are the sum of $\mathrm{fix}(\beta)$, summed over all reflections β. Here are the expressions for $\mathrm{fix}(\beta)$ that I found. In the case ℓ and n both odd, all M reflections are essentially $\beta(x) = -x \pmod{M}$. In the other three cases, $M/2$ are of that form, and the other $M/2$ are of the form $\beta(x) = 1 - x \pmod{M}$.

parity of n	parity of ℓ	parity of r	$\beta(x) = -x$	$\beta(x) = 1 - x$
odd	odd	even	$\mathrm{fix}_{\mathrm{r}}(\beta) = \binom{\frac{n-1}{2}}{\frac{r}{2}}\ell^{\frac{r}{2}}$	———
odd	odd	odd	$\mathrm{fix}_{\mathrm{r}}(\beta) = \binom{\frac{n-1}{2}}{\frac{r-1}{2}}\ell^{\frac{r-1}{2}}$	———
odd	even	even	$\mathrm{fix}_{\mathrm{r}}(\beta) = \binom{\frac{n-1}{2}}{\frac{r}{2}}\ell^{\frac{r}{2}}$	$\mathrm{fix}_{\mathrm{r}}(\beta) = \binom{\frac{n-1}{2}}{\frac{r}{2}}\ell^{\frac{r}{2}}$
odd	even	odd	$\mathrm{fix}_{\mathrm{r}}(\beta) = 2\binom{\frac{n-1}{2}}{\frac{r-1}{2}}\ell^{\frac{r-1}{2}}$	$\mathrm{fix}_{\mathrm{r}}(\beta) = 0$
even	odd	even	$\mathrm{fix}_{\mathrm{r}}(\beta) = \left[\binom{\frac{n-2}{2}}{\frac{r}{2}} + \ell\binom{\frac{n-2}{2}}{\frac{r-2}{2}}\right]\ell^{\frac{r-2}{2}}$	$\mathrm{fix}_{\mathrm{r}}(\beta) = \binom{\frac{n}{2}}{\frac{r}{2}}\ell^{\frac{r}{2}}$

even	odd	odd	$\mathrm{fix}_{\mathrm{r}}(\beta) = 2\binom{\frac{\mathrm{n}-2}{2}}{\frac{\mathrm{r}-1}{2}}\ell^{\frac{\mathrm{r}-1}{2}}$	$\mathrm{fix}_{\mathrm{r}}(\beta) = 0$
even	even	even	$\mathrm{fix}_{\mathrm{r}}(\beta) = \binom{\frac{\mathrm{n}-2}{2}}{\frac{\mathrm{r}}{2}}\ell^{\frac{\mathrm{r}}{2}}$	$\mathrm{fix}_{\mathrm{r}}(\beta) = \binom{\frac{\mathrm{n}}{2}}{\frac{\mathrm{r}}{2}}\ell^{\frac{\mathrm{r}}{2}}$
even	even	odd	$\mathrm{fix}_{\mathrm{r}}(\beta) = 2\binom{\frac{\mathrm{n}-2}{2}}{\frac{\mathrm{r}-1}{2}}\ell^{\frac{\mathrm{r}-1}{2}}$	$\mathrm{fix}_{\mathrm{r}}(\beta) = 0$