

Honors 1483
Mathematics of Games and Politics
Syllabus

Fall 2008

Instructor: Sam Smith, Mathematics

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Office Hours: Mon 11:00-1:00, Tues. 1:00-3:00, or by appointment.

Text: Sam Smith, *The Mathematics of Games of Pure Chance and Games of Pure Strategy*, available at www.sju.edu/~smith/Current_Courses/games.pdf

Supplementary Articles: These articles are all available (and free!) on JSTOR

1. Amartya Sen, *The impossibility of a Paretian liberal*, *Journal of Political Economy*, vol 178 1970, 152-157
2. Alfred F. Mackay, *A simplified proof of an impossibility theorem*, *Philosophy of Science*, Vol 40, 1973, 175-177
3. Hannu Nurmi, *Voting procedures: a summary analysis*, *British Journal of Political Science* Vol 13, 1983, 181-208
4. Barry O'Neill, *International escalation and the dollar auction*, *Journal of Conflict Resolution*, 1986, 33-50

Course Description: The study of games as a source of purely mathematical problems dates back to the gambling halls of Paris in the 1650s. Basic questions about odds challenged the best mathematicians of the day, including Blaise Pascal and Pierre de Fermat, to invent the modern theory of probability. In the early twentieth century, physicists discovered evidence that sub-atomic particle interactions are not deterministic but are best described with probabilities. Quantum mechanics and the uncertainty principle made probability theory an indelible feature of our world view. While physicists were pondering "Schrödinger's Cat", social scientists began using simple games to model social and political interactions. The game of "Chicken" stood as a powerful and frightening metaphor for the nuclear arms race in the 1950s. The game "Prisoner's Dilemma" gave a concise and utterly confounding picture of human interactions. The problems and implications of these and other simple games have remained a focal point in the study of human behavior. Meanwhile, with its theoretical underpinnings provided by two remarkable theorems, one due to John von Neumann and the other to John Nash, game theory emerged after the second world war as a genuine mathematical discipline with important applications in economics and social thought. In the 1950s, Kenneth Arrow initiated the mathematical study of elections or Voting Theory using similar

ideas and techniques as those in game theory. By considering the issues of strategy and fairness which arise in social choice, Arrow formulated and proved his famous Impossibility Theorem, for which he won the Nobel Prize in 1972. Subsequently, Martin Shubik, Lloyd Shapley and others put ideas familiar from probability theory to work to analyze the power of individuals in a voting system.

In this course, we will trace a path through the historical development of the ideas sketched above and develop the mathematics necessary to understand the techniques and the implications of the results. While we will be focused, in each case, on the mathematical aspects of the theory, this will not be our exclusive focus. We will have many political questions, paradoxes and open problems to consider which will take us out of the mathematical realm. Assignments in the course will be balanced between mathematical problem solving and written analysis.

Learning Goals: Students will learn to compute probabilities and expected values using counting techniques. Students will learn to compute conditional probabilities using probability trees. Students will learn the definition of Nash equilibria and be able to determine these in two player games. Students will be able to compute mixed strategy equilibria in a two player, two strategy game. Students will be able to implement voting methods and determine whether these satisfy or violate fairness criteria. Students will be able to compute power indices for yes/no voting systems and determine whether a given yes/no voting system is weighted. Students will learn the meaning of theorem and proof.

Course Expectations: This course is designed to be interactive. Lectures will frequently involve experiments (games!) and discussions. Thus your attendance and participation in this class are crucial. We will have various written assignments and readings as scheduled on the Course Calendar. We will have a Midterm Exam on *Thursday, October 16* and a cumulative Final Exam given at the date scheduled by the registrar.

Grades: I will determine your final course grades by curving your total scores, out of a possible 750 points, computed as follows: The Midterm exam will be worth 100 points and the final exam 200 points. The written assignments will be worth a total of 400 points. I will award (a maximum of) 50 points for class attendance and participation.

The Assignments: Written assignments will be generally of two types. We will have several mathematics exercise sets in which you will solve problems. We will also have an essay assignment in which you will give your written analysis of various topics from the course. The standards for grading these two types of assignments will be very similar, however. When presenting solutions to a mathematical problem, I will expect you to write clearly and in complete sentences. In fact, many of the mathematical exercises will have a discussion component. In the essay, on the other hand, I will expect you to incorporate

mathematical ideas and, perhaps, even calculations to help argue or illustrate your points.

Policy on Collaboration: You are welcome and encouraged to work with others in the class and, of course, to ask me questions. However, the work you hand in must be exclusively your own. A good rule of thumb for mathematical problems, is to work with others on solving a problem but to write up your solution by yourself.

Policy on Academic Honesty: I will adhere to the Academic Honesty Policy as stated in the University Catalogue. All tests and assignments found to be in violation of this policy will receive a zero.

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Monday	Thursday
Sept 1 No Class	Sept 4
Sept 8 Read Smith: Sections 1, 2, 3	Sept 11
Sept 15 Read: Smith: Section 4	Sept 18 Probability Exercises Sections 2, 3 Due
Sept 22 Read: Smith: Section 5	Sept 25 Probability Exercises Section 4 Due
Sept 29 Read Smith: Section 6	Oct 2 Probability Exercises Section 5 Due
Oct 6 Read Smith: Section 7	Oct 9 Game Theory Exercises Section 6, 7 Due
Oct 13 Read: Smith: Section 8	Oct 16 Midterm Exam
Oct 20 Fall Break -- No Class	Oct 23
Oct 27	Oct 30 Game Theory Exercises Section 8 Due
Nov 3 Read Nurni Sections 1 through 4	Nov 6
Nov 10 Read Nurni Section 5	Nov 13 Weighted Voting Theory Exercises Due
Nov 17 Read Mackay	Nov 20
Nov 24	Nov 27 Thanksgiving -- No Class
Dec 1 Read Sen	Dec 4 Social Choice Exercises Due
Dec 8	Dec 11 Social Choice Essay Due
Dec 15 Final Exams	Dec 18