# MATH 2091
# Real Analysis
# Syllabus

Fall 2008

**Instructor:** Sam Smith
**Office:** Barbelin 230, email smith@sju.edu, x1559
**Office Hours:** Mon 11:00-1:00, Tues. 1:00-3:00, or by appointment.
**Text:** Abbot, Understanding Analysis, Springer-Verlag
**Course Description:** The discovery and development of the calculus represents one of the fundamental achievements. of mathematics as well as one of its most richly historical stories. With beginnings in the method of exhaustion by limits of Archimedes, the development of calculus traces through the tangent line computations of Fermat, the publication of Newton's *Principia*, the early, remarkable applications for physics achieved by Newton and his contemporaries, through substantial depth-changes in the hands of Riemann, Cauchy, Weierstrass, Cantor and Lebesgue to the present. While the methods of calculus provided mathematicians with unprecedented computational power, the theoretical issues inherent in the subject required a first direct confrontation with the *horror infiniti*, the fear of infinity that characterized much of mathematics dating from the Greeks. Considerations of infinite processes – sequences, limits and series -- entailed a major paradigm shift from the geometry and number theory studied classically. It is not surprising that mathematicians made false steps and only slowly developed the proper language for calculus. While the study of real-valued functions flourished in the years following Newton, the lack of rigorous underpinnings for the subject and the strong reliance on intuition instead of formal proof eventually led to foundational problems. Faced with contradictory results, nineteenth century analysts led by Cauchy and Weierstrass revamped the subject by giving precise definitions to the most basic terms like functions, limits and continuity. Most notably, these mathematicians formulated the famous $\varepsilon$–$\delta$ definition, establishing a new standard of rigor for the subject and, by extension, for all of mathematics. This increased rigor, in turn, led to a host of deeper, more profound questions and results about the nature of the real numbers. For example, Cantor's theory of cardinality of sets, a measure theory of infinities, emerged from his work on the analysis of the real numbers. Cantor's work, in turn, set the stage for Gödel's impossibility theorems and other amazing developments in twentieth century mathematical logic. In analysis, his results were extended by Baire to a fundamental characterization of the "size" of the set of real numbers with applications to functional analysis and by Lebesgue to a modern theory of integration.

In this course, we take up the study of real analysis, the rigorous study of functions of a real variable, following much of the historical path through the material described above. Many of the concepts and results of this course will be familiar from calculus. We will study continuity, differentiability, convergence of sequences and series and integrals. In contrast to your calculus classes, however, our approach to this material will be theoretical. We will be focused on the proofs! Moreover, we will consider a variety of general questions beyond the scope of a non-theoretical course. Our inquiry will be example-driven, with highlights including the Dirichlet and Thomae functions, Cantor's "middle-thirds" set and Weierstrass' everywhere continuous but nowhere differentiable function. We will also consider a whole new class of questions which emerge from but go far beyond the scope of introductory calculus, including the nature of sets of discontinuities, continuity properties of derivatives and, as time permits, integrability of functions with discontinuities. We will cover the first six or seven chapters of the text.

**Course Structure:** Your responsibilities for this course are: a midterm exam, a final exam, ten written problem sets and at least one in-class presentations (see below). The dates for these are indicated on the attached course calendar except for the final exam which will be scheduled by the registrar.

**Learning Goals**: Students will be able to prove convergence and divergence of limits using the $\varepsilon-\delta$ definition. Students will be know and be able to prove basic theorems about the notions of completeness, compactness and connectedness. Students will know and be able to prove basic facts about derivatives and their properties. Students will know and be able to prove basic facts about infinite series of functions. Students will know the definition of the Riemann integral and how to compute this from the definition in elementary cases.

**Problem Sets:** I will hand out problem sets essentially every week. Problems will be of two types: *practice problems*, which will reinforce the basic concepts and methods of proof and *further problems*, whose solutions will usually be longer and more involved. The practice problems will be worth a total of 30-40 points and the further problems will be worth anywhere from 5 to 15 points each. I will collect problem sets on Thursdays (see the attached calendar). *I will not accept practice problems after the due date.* Thus you should hand in as many of these as you have solved. I will accept further problems up to a week late with appropriate point reductions. Further problems which have not been solved after a weeks time will become *open questions*. I will accept solutions to open problems at any time. The submission of a correct solution to an open problem will, of course, render the problem closed. Solutions to most of the problems in this course will be *proofs*. I expect you to write proofs in complete, grammatical sentences (albeit with symbols). You will lose substantial points if your work is not neat, well organized and written in complete, sentences.

**Grades:** The minimal requirements for this course are that you hand in practice problems every week give at least one presentation and do passing work on the exams. To get a B in this course you should solve virtually all the practice problems each week, attempt and solve, on average one further problem a week, give presentations and do reasonably well on the exams. For an A, you should solve virtually all the practice problems, a large percentage of the further problems and give one or two presentations of unique solutions. Your performance on the tests should indicate mastery of the material.

**Policy on Collaboration:** You are encouraged to discuss the ideas of the class with fellow students or with me. However, I expect every student to hand in their own work. If you have gotten considerable help from another (excluding me) you should indicate this on your paper.

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# Calendar

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| Monday | Thursday |
|---|---|
| Sept 1 | Sept 4 |
| Sept 8 | Sept 11<br>**Problem Set 1 Due** |
| Sept 15 | Sept 14<br>**Problem Set 2 Due** |
| Sept 22 | Sept 25<br>**Problem Set 3 Due** |
| Sept 29 | Oct 2<br>**Problem Set 4 Due** |
| Oct 6 | Oct 9<br>**Problem Set 5 Due** |
| Oct 13 | Oct 16<br>**Midterm Exam** |
| Oct 20<br>**Fall Break -- No Class** | Oct 23 |
| Oct 27 | Oct 30<br>**Problem Set 6 Due** |
| Nov 3 | Nov 6 |
| Nov 10 | Nov 13<br>**Problem Set 7 Due** |
| Nov 17 | Nov 20<br>**Problem Set 8 Due** |
| Nov 24 | Nov 27<br>**Thanksgiving -- No Class** |
| Dec 1 | Dec 4<br>**Problem Set 9 Due** |
| Dec 8 | Dec 11<br>**Problem Set 10 Due** |
| Dec 15 | **Final Exams** |