

Math 132
Mathematics of Games and Politics
Syllabus

Spring 2012

Instructor: Sam Smith, Mathematics

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Office Hours: Mon 11:00-1:00, Tue 1:00-4:00 or by appointment.

Text: Sam Smith, **Chance, Strategy and Choice**, available for purchase at the University Press.

Course Description: The study of games as a source of purely mathematical problems dates back to the gambling halls of Paris in the 1650s. Basic questions about odds challenged the best mathematicians of the day, including Blaise Pascal and Pierre de Fermat, to invent the modern theory of probability. In the early twentieth century, physicists discovered evidence that sub-atomic particle interactions are not deterministic but are best described with probabilities. Quantum mechanics and the uncertainty principle made probability theory an indelible feature of our worldview. While physicists were pondering “Schrödinger’s Cat”, social scientists began using simple games to model social and political interactions. The game of “Chicken” stood as a powerful and frightening metaphor for the nuclear arms race in the 1950s. The game “Prisoner’s Dilemma” gave a concise and utterly confounding picture of human interactions. The problems and implications of these and other simple games have remained a focal point in the study of human behavior. Meanwhile, with its theoretical underpinnings provided by two remarkable theorem, one due to John von Neumann and the other to John Nash, game theory emerged after the second world war as a genuine mathematical discipline with important applications in economics and social thought. In the 1950s, Kenneth Arrow initiated the mathematical study of elections or *social choice theory* using similar ideas and techniques as those in game theory. By considering the issues of strategy and fairness that arise in social choice, Arrow formulated and proved his famous Impossibility Theorem, for which he won the Nobel Prize in 1972.

In this course, we will trace a path through the historical development of the ideas sketched above and develop the mathematics necessary to understand the techniques and the implications of the results. While we will be focused, in each case, on the mathematical aspects of the theory, this will not be our exclusive focus. We will have many political questions, paradoxes and open problems to consider which will take us out of the mathematical realm.

Learning Goals: Students will be able to construct truth-tables for logical statements. Students will be able to construct a game tree for a strategic game and use this to solve the game. Students will learn to compute probabilities and expected values using counting techniques. Students will learn to compute conditional probabilities using probability trees. Students will learn the definition of Nash equilibria and be able to determine locate these in specific games. Students will be able to compute mixed strategy equilibria in a two-player, two-strategy game. Students will be able to implement voting methods and determine whether these satisfy or violate fairness criteria. Students will learn the meaning of theorem and proof. Students will know the details of the proof of several theorems including: the 2x2 Minimax Theorem, the equivalence of various auction methods, O'Neill's Theorem for the Dollar Auction, the classification of social dilemmas and some impossibility theorems in social choice theory.

Course Expectations: This course is designed to be interactive. Lectures will frequently involve experiments (games!) and discussions. Thus your attendance and participation in this class are crucial. We will have three written assignments and four quizzes as scheduled on the attached Course Calendar. We will have a midterm exam on *Thursday, March 1* and second exam on *Thursday, April 19*. The cumulative final exam will be given at the date scheduled by the registrar.

Policy on Absences: All quiz and exam dates are *mandatory attendance*. Make-up tests and quizzes will only be given for students who contact me 24 hours before the missed class with a legitimate reason for the absence. In all cases, the missed test or quiz must be taken before or during the next class period.

Grades: I will determine your final course grades by curving your total scores, out of a possible 710 points, computed as follows: The exams will be worth 100 points each and the final exam 200 points. The written assignments will each be worth 50 points and the quizzes will each be worth 40 points.

Lecture Schedule and the Homework Assignments: The attached Course Calendar gives the (tentative) schedule for the course. I will lecture on each chapter in the text consecutively and generally cover one or two per week. You are expected to work on the exercises for each chapter as they are covered. I will collect homework assignments three times during the semester. You will hand in exercises you have completed or attempted from the chapters covered to date. There will be some possibilities for extra credit here for work on harder problems. I will be happy to talk about exercises during class or in office hours if you have questions. The exercises in the text will form the basis for the problems on quizzes and exams.

Policy on Academic Honesty: I will adhere to the Academic Honesty Policy as stated in the University Catalogue. All tests and assignments found to be in violation of this policy will receive a zero.

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Monday	Thursday
Jan 16 No Class: MLK Day	Jan 19 Read Chapter 1
Jan 23 Read Chapter 2	Jan 26
Jan 30 Read Chapter 3	Feb 2 Quiz 1
Feb 6 Read Chapter 4	Feb 9 Read Chapter 5
Feb 13 Read Chapter 6	Feb 16 Quiz 2 Assignment 1 Due
Feb 20 Read Chapter 7	Feb 23 Read Chapter 8
Feb 27	Mar 1 Midterm Exam
Mar 5 No Class: Spring Break	Mar 8 No Class: Spring Break
Mar 12 Read Chapter 9	Mar 15 Read Chapter 10
Mar 19 Chapter 11	Mar 22 Quiz 3 Assignment 2 Due
Mar 26 Read Chapter 12	Mar 29 Read Chapter 13
Apr 2 No Class: Easter Break	Apr 5
Apr 9 Read Chapter 14 Quiz 4	Apr 12 Read Chapter 15
Apr 16 Read Chapter 16	Apr 19 Test 2
Apr 23 Read Chapter 17	Apr 26 Assignment 3 Due
Apr 30 Final Exam Week	May 3